Project Presentation

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Summary: This is a qualitative study on differential intervention applied as a research methodology that puts the teacher in the role of teacher-researcher. The project inserts itself in the international scope of present-day higher-education didactics research, which reports on the difficulties engineering students face due to the *triple discontinuity* problem. This problem will be addressed in our Calculus I classroom for Computer Engineering freshmen students. The following question is posed as a starting point for the research: *How to create an environment favorable to the simultaneous construction of three bridges in order to address the triple discontinuity?* The classroom is seen as a micro-institution that is part of the ideological state apparatuses of the capitalist social formation, in which *know how* refers to concepts, techniques, behavioral norms, and scientific language, as well as to being aware of the process that produces the qualified labor force. We expect the establishment of such an environment to follow the principle of *one teaches by listening, and one learns by talking*, which is founded on a theoretical framework grounded on Hegel, Marx, and Lacan. Under this framework, we redefine the concepts of learning, didactics, and pedagogy; we further propose the *formative-summative-informative assessment model* as a means to oppose the social segregation inherent in the current traditional teaching model.

Keywords: Calculus teaching; Mathematical education for engineering; Teacher-researcher; Triple discontinuity; Formative-summative-informative assessment.

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Introduction

The preoccupation with the exact sciences in higher education dates back to the creation of the *International Commission on Mathematical Instruction* (ICMI) in 1908, which suggested, in a 1911 meeting, what mathematics should be taught to physics and natural sciences students (HOWSON et al., 1986). In 1986-88, the ICMI conducted a new study on the status of professional mathematics teaching in various higher-education institutions around the world. That study centered on critical questions about teaching mathematics in programs such as engineering: "At the International Congress of Mathematicians held the following year, in 1912, there was a discussion on mathematics for engineers, and **who** should best teach it." (HOWSON et al., 1986, our highlight). Similar questions were compiled by Frota and Nasser (2009) when reporting on the state of the art of mathematic courses in Brazilian higher education: "What is the role of mathematics in higher education? How do students relate to formal mathematics? How should this mathematics be approached? What strategies do students employ to learn mathematics?" (p.7, 2009).

Research topic

The topic, or **subject**, of our investigation refers to the teaching of mathematics to engineering students, especially calculus. The question mentioned in Howson et al. (1986) about who should teach mathematics to engineers is very delicate, and reflects the long-time dissatisfaction of the engineering faculty with the teaching of disciplines such as calculus. These courses are usually taught by professors whose education is in pure mathematics: "Traditionally, basic sciences, physics and chemistry, and mathematics are required as core subjects for engineering education and have been taught independently by faculty members from mathematics and basic sciences" (QUINTANILLA et al., 2007, p.1).

This separation between mathematics and engineering therefore constitutes an epistemology issue and requires that the teaching of these mathematics courses be understood.

It is known that most of the professors who teach mathematics in engineering programs were trained as researchers; in other words, teaching was not the primary focus of their education. This education in pure mathematics includes, at least in Brazil, three years in an undergraduate bachelor's program, where one learns to deal with theories based on definitions, theorems, and demonstrations. The crowning of that effort leads to a doctorate degree in a very specific topic, and marks the moment when the student is finally apt to produce their own theorems and proofs based on recent knowledge. Therefore, higher-education mathematics professors are taught to ensure the axiomatic form when presenting mathematics through exposition, and that is what they tend to reproduce when teaching, which makes the learning experience for the engineering student much more difficult. This concept is also mentioned by Christensen (2008):

> Presenting new mathematics to students in this way, in the form of lectures, is one framework for the learning of mathematics. However, this approach suffers from at least one problem, namely that it can be quite difficult to connect the abstract formalism of mathematics with the necessary applicable skills in a given profession (p. 131),

which expresses a clear preoccupation with the gap that arises when mathematics is taught to students from other areas: "This may result in a gap in the students' ability to use mathematics in their engineering practices; it might even be the case that no matter how much abstract mathematics we teach engineering students, it will not help bridge this gap." Christensen (2008, p.131).

The implications are well known: students are faced with an increasing number of failing grades due to their insufficient performance, especially in calculus, and eventually drop out of their chosen field altogether (BARUFI, 1999; ELLIS; KELTON; RASMUSSEN, 2014; REZENDE, 2003; TALL, 1992).

The students also perceive as evident the disconnection between what they are taught – concepts, definitions, theorems, demonstrations – and problems related to their chosen profession (GONZALEZ-MARTIN, 2018):

> It has been observed that students have long complained about calculus, and consider it abstract and unnecessary. Institutional evaluations report that few students have the "proficiency" to deal with the concepts presented in the course. Few state having any interest in studying the subject (CABRAL e CATAPANI, 2003).
>
> We can say the same applies in Brazil, where physics and mathematics courses were developed within the logic of these sciences, **without an effort to integrate them with practical professional applications.** This effect is but one example of the **resistance shown by both faculty and academic community to in-depth changes in the curriculum** [...] (SILVEIRA, p. 21, 2005, our highlight).

We must note that as teachers decide which concepts to teach, they do not consider the concepts' full history, from inception to being endorsed by the scientific community. From the perspective of didactic transposition (CHEVALLARD, 1991), a scientific concept, the *scholarly knowledge*, must be transformed by the teacher before it can, through negotiation and evaluation of the students' prior notions, be finally taught:

> Content that has been designed as *taught knowledge* undergoes a series of adaptive transformations that enables it to take its place among the *objects of learning*. The "work" that changes an object of the knowledge to be taught into a learning object is called *didactic transposition*[1] (CHEVALLARD, 1991, p. 45, author's emphasis, our translation).

In addition to the difficulty inherent in learning calculus concepts, this further contributes to students dropping the courses and to increased difficulties in dealing with elementary mathematics. These difficulties, called "gaps in basic education", become evident when analyzing the students' answers to problems or exercises that require skills in arithmetic, geometry, trigonometry, and algebra. In higher education, there have been long-standing efforts to compile, describe, and suggest courses of action to overcome misconceptions in elementary mathematics (BALDINO and CABRAL, 1999; CABRAL, 2001; CURY, 2004, 2008; MOVSHOVITZ-HADAR; ZASLAVSKY; INBAR, 1987; NASSER, 2009; RADATZ, 1979).

The situation becomes even more complex when one considers that mathematical justification at the elementary level is relatively distinct from that required in higher education, as has been shown by some of the early research on Advanced Mathematical Thinking (DREYFUS, 1991; DUBINSKY, 1991; GRAY et al., 1999; TALL, 1991). However, we must note that, according to Tall (1991), there are characteristics of advanced mathematical thinking present in the learning process even in basic education: “The problem-solving procedures of entry, attack and review can *and are* being performed by younger children in such mathematical investigations. Thus, many of the processes of advanced mathematical thinking are already found at a more elementary level” (TALL, 1991, p. 20, author's emphasis).

The teacher must therefore face two simultaneous difficulties when broaching the moment of passage from basic to higher education. The solution found by many institutions to deal with these problems is to push the first calculus course up on the grid, thus delaying the students' first contact with the subject and instead enrolling them in a class designed “to fill gaps in knowledge, to have students review (or study)

[1] Original em espanhol: Un contenido de saber que ha sido designado como saber a enseñar, sufre a partir de entonces un conjunto de transformaciones adaptativas que van a hacerlo apto para ocupar un lugar entre los *objetos de enseñanza*. El “trabajo” que transforma de un objeto de saber a enseñar en un objeto de enseñanza, es denominado la *transposición didáctica*.

essential facts and skills that have been forgotten, concentrating on those topics that are essential for their first year mathematics courses" (WOOD, 2002, p. 90).

This measure is evidence that the teaching of mathematics, as well as physics and many other subjects, is still contingent upon the structure of specific departments. This is an incorrect response by the institution, and shows its resistance in adapting the epistemological concepts involved in teaching to an engineering program. After all, the same work with variation rates and their accumulations can be developed in a high school mathematics course.

The overall setting of higher education, and especially in teaching calculus to engineering students, presents an abundance of challenges. Despite the impact of the pandemic on the world's economy, the current socioeconomic framework still defines and sets clear trends for both academic education and technical training. The technology labor market has weathered the crisis and continues to grow, demanding cross-disciplinary interventions and highly qualified education, thereby pressuring students to continue their studies into masters and doctorate programs. This continued education is often encouraged or required – and sometimes even supported –, by companies.

The problem

In short, the **problem** identified lies in the gaps that can be observed at specific moments in the education of a professional. Viirman (2022), and Winsløw and Grønbæk (2014) report that as early as in 1908 Felix Klein had called attention to a *double discontinuity* in the education of mathematics teachers: (1) in the transition students undergo upon leaving basic education and entering higher education, and (2) when they finish their higher education and return to basic education as teachers.

These discontinuities cannot be ignored and are understood, up to this point, as constituting a process in which there is no sublation between:

1. basic and higher education,
2. calculus courses and technical engineering courses, and
3. higher education and the labor market.

In the literature on higher education didactics discussed in this study we have found that authors characterize three discontinuities, which Florensa et al. (2022) call the *triple discontinuity*. Based on the Anthropological Theory of the Didactic (ATD), and grounded on the concepts of *institution* and *institutional position*, the authors examine the role of mathematics in engineering programs, which allows them to identify the *internal discontinuity* faced by the students of a given institution as they move from mathematics classes to engineering classes. Florensa et al. (2022) thereby adds a third discontinuity to Felix Klein's conjecture of the double discontinuity.

According to the principles and concepts of the theory we have developed, based on Hegel, Marx, and Lacan, we will approach this *triple discontinuity problem* from the perspective of our own classroom of calculus for computer engineering students. We view our classroom as a *micro-institution* that is part of the *ideological state apparatuses of the capitalist social formation* in which "One thereby learns the 'savoir faire'"[2] (ALTHUSSER, 1995, p. 273, our translation), as well as the concepts, techniques, scientific language, behavioral rules, and awareness of the functioning of the school system (CABRAL; BALDINO, 2019) in which the process for producing qualified labor-power for the labor market takes place.

Therefore, the question on which to ground our research is: *How to create an environment favorable to the simultaneous construction of three bridges in order to address the triple discontinuity?* This is a place in which students will endeavor to: (1) solve problems involving signifiers related to their chosen field – electric-electronic engineering, (2) relate those signifiers to mathematical signifiers – calculus and basic math, and (3) identify the signifiers as pertinent to the profession of computer engineer.

Hypotheses

We propose that an environment that allows students to overcome the three discontinuities can be developed by combining didactics, pedagogy, and epistemology, when we take into account: (1) an observation of the students to determine their preferred methods of justification, and (2) the non-linear organization of knowledge, or *signifier network.* This environment, in which the student is encouraged to interact and dialog with colleagues and teachers, must allow the student to become a professional by experiencing the *effect of the return of his speech.* It is in such an environment, where the student is required to systematically employ the signifiers in all three positions, that the *professional object* must be developed.

Objectives

The objective is to examine the possibility of building bridges to address the *triple discontinuity*, taking into account didactic and pedagogical aspects grounded on the epistemological model whose principle is that *the subject is constituted by*

[2] Original in French: "On apprend donc des «savoir faire»".

representing itself to a signifier by means of another signifier. The subject, denoted by $\not{S}$ and defined as an effect of the signifier, is what a signifier represents to another signifier" (LACAN, 2011, p. 206, our translation). Regarding the didactic aspect, the goal is to produce worksheets comprising activities and problems related to the professional fields. The objective is to (1) identify and address the students' difficulties with basic education content while they deal with new concepts; (2) lead students to question, explain, and defend their reasoning on the solutions they propose to problems studied; and (3) encourage and promote dialogs about the requirements and conditions for joining the workforce, both by introducing specific problems and based on the students' commitment to learning – the professional ethics of school education. In regard to epistemology, the goal is to provide a preview of concepts and content from the signifier network of the profession, grounded on the infinitesimal conception of calculus and on the concept of *identity-quilted speech* (BALDINO; CABRAL, 2021).

Justification

The project's proposal is aligned with recent research into *didactics in higher education*, especially concerning the teaching of calculus to engineering students. We have taken into consideration both research published in international journals and works presented at international events. Among the scientific events, of note are the International Network for Didactic Research in University Mathematics (INDRUM) and the Congress of the European Society for Research in Mathematics Education (CERME). Specifically, we recently participated in and presented our work (BALDINO; CABRAL, 2022), at TWG 14 – University Mathematics Education, CERME 12, which was held online via Zoom Meeting. At this last event, we had fruitful exchanges with participating researchers, such as Ignasi Florensa, Olov Viirman, Carl Winsløw, and Alejandro González-Martín, who have been investigating how the work might be reformulated based on what takes place in teaching mathematics in higher education, especially in calculus courses, so as to address the discontinuities faced by students in their professional education. The attempts to bridge the discontinuities show that these moments of passage are, in a way, treated separately. The idea we are suggesting be developed is to establish an environment that allows for experimenting with building the three bridges simultaneously.

Adequate conditions for carrying out this proposal are in place. The Computer Engineering program has its own dedicated teaching staff – there are no external departments specializing in the basic sciences. It should therefore be easier to foster conversations between teachers with different backgrounds in order to drive collaboration and the exchange of ideas about how the work must be carried out in the classroom.

One of the researchers has a degree in electrical engineering and a doctorate in pure mathematics, a skill set that allows for developing assets for use in the classroom that are closer to real-world problems in electromagnetism. The other researcher has a degree in mathematics education, a doctorate in education, and, having specialized in mathematics education and psychoanalysis, is well prepared to deal with students' challenges in moving from basic to higher education, and their difficulties with subjectivity. In addition, the students enrolled in the classes are normally computer engineering majors, and only very rarely come from other programs.

Theoretical framework

Mathematics teachers, whether hailing from education or mathematics backgrounds, find it challenging to teach courses to engineering students. The work we mentioned previously emphasizes that teaching calculus – and even other mathematics disciplines – to engineering students requires adaptations in terms of both didactics and pedagogy (CHRISTENSEN, 2008; GONZALEZ-MARTIN, 2018), in addition to breaking with the epistemological view of current traditional teaching (CTT), which we characterize and criticize in Cabral, Pais, and Baldino (2019), and in Cabral (2015).

As we mentioned previously, in the last few decades we have established principles and concepts under a theoretical framework grounded on Hegel, Marx, and Lacan, in order to support didactic and pedagogical practices. We highlight the following concepts:

Learning is representing the subject through new signifiers. This representation must finally and necessarily take place in everyday language. We suggest that these signifiers – concepts, definitions, etc. – are pertinent to three registers: *phonemes, graphemes, and mathemes.* When a student takes the position of speaker, they risk a new representation in the domain of the Other, where they must constitute themselves as a desiring being. Learning is talking, and, in a broad sense, that includes gestures, hesitations, and silence. It is a restructuring of the subject's *jouissance* that depends on myriad factors, like cultural background, gender, ethnicity, etc. The moment that learning takes place cannot be determined beforehand; it usually takes place overnight. We say that it "cannot be observed *in vitro*." Oftentimes, when we're starting a class that we designed to solve a difficulty the students manifested in the previous class, we are surprised to find that the difficulty is gone and that they are using the new signifiers adequately.

Didactics is understood as the sequential treatment – in the chronological time of a sequence of classes – of learning tasks that consider the students' responses to prior learning tasks, a fact that supports its connection to the evaluation model, which we define further on. *Pedagogy* is understood as concerning the work of monitoring the students' connection to the learning task, considering its direct relationship with the current summative assessment model that determines whether or not the students will receive course credits. In current traditional teaching, pedagogy is customarily centered on expository classes in lecture halls (*séance magistrale*) followed by workshops for smaller groups of students; the process culminates in exams to assess what was learned – in other words, to check whether students will represent themselves in writing by the new signifiers that were the object of the course. We argue that this model is profoundly unfair, since it assumes that equal didactic conditions equate with equal opportunities to learn. The institution thus exempts itself of responsibility for the students' learning, and they are left to learn on their own, under penalty of not passing the course.

The model of pedagogy and didactic we are proposing, on the other hand, seeks to *treat different people differently*. It aims to both compensate for the social segregation rooted in the students' cultural backgrounds, and to leverage remote education, a contingency of the pandemic, in order to assess and guide the dialogs in the classroom. Our model intends to ground summative assessment on the understanding of texts and on problem-solving before trying to determine how far the students can develop solutions on their own. It is about *rewarding the effort to learn* (BALDINO; CABRAL, 2021).

According to Luckesi (2011), evaluation practices evolved, over the years, into assessment models aimed at diagnosing the results of the educational measures used. These models were developed to answer the requirement of effective quality in the returns of investments in education, especially in the United States. In short, the author explains that the various evaluation concepts lead to different assessment models that are tailored to suit specific moments in the school semester. Each assessment model contains a set of tools for collecting data about teaching processes and learning.

Based on the premises and work intentions described thus far, which are grounded in Hegel, Marx, and Lacan, we define the *formative-summative-informative assessment model,* having distinct objectives. *Formative assessment* is conceived as a continuous process across interactions that aims to listen, welcome, interpret, and turn the students' answers into the object of work. It is closely related to didactics, as it informs the decisions regarding what teachers must do from one class to the next, given the specific level of demand that students can tolerate on their efforts. *Summative assessment* is delimited by the non-mathematical criteria that determine whether students are allowed to progress in the institutional credit system. The students' effort to understand is rewarded, as opposed to assessment that rewards the amount learned at the end of the course. *Informative assessment* consists in a declaration to the institution about how each student completed their learning journey in the course over the semester, informed by a final test. The *informative assessment* validates our summative assessment model based on effort to understand.

The underlying principle of our educational practice is *one teaches by listening, and learns by talking* (BALDINO; CABRAL, 2022; CABRAL, 2005; CABRAL; BALDINO, 2010; CABRAL; PAIS; BALDINO, 2019), a counterpoint to current traditional teaching (CTT). This is the necessary condition for accessing the students' preferred methods of justification. Lacan (1973) teaches about the importance of speaking as a means to access the unconscious, which is structured as language and manifests itself through jokes, metaphors, metonymies, truth, doubt, certainty, etc. Therefore it is essential, in the classroom, to put students in the role of speakers, enabling the suspension of their mutism, following the example of the analytical experience: "It is not up to the analysis to find in a case the differential trait of the theory and believe this trait will serve to explain why your daughter is mute – because what is at play is making her speak"[3] (1973, p. 15, our translation), such that we may get closer to: (1) their difficulties, (2) their means of justification, (3) their perceptions about the relationship between mathematics and their college program, and (4) their beliefs about how the course is related to their chosen professions. In Lacan, the unconscious opens up, albeit temporarily, through Freudian slips and jokes in speech, allowing access to the enigma of the subject. This is where the language of the

[3] Original in French: Mais l'analyse n'est pas de retouver dans un cas le trait différentiel de la théorie, et de croire expliquer avec pourquoi votre fille est muette – car ce dont il s'agit, c'est de *la faire parler*.

unconscious manifests itself. And Lacan aims to act on subjectivity by accessing the *signifier network.*

It is important to note that remote education proved to be a facilitator for the underlying principle we mentioned before. Remote education makes eye-on-eye conversations possible, especially between the teacher and each student. This allows the teacher to observe the students while they talk – their mannerisms, expressions of surprise and disappointment, the doubt on their faces when their certainties are shaken. From the students' perspective, they are facing someone impassive. The teacher doesn't show any surprise at the answers and justifications he hears. The teacher simply and genuinely listens. This genuine listening is selective, as it is restricted to what the students are expressing in their *quilting point* work, allowing the teacher to give them other questions and instill other doubts. Genuine listening, despite being restricted to mathematics, brings out the same temporal phenomenon that takes place at the clinic, that is, the *logical time*, established by the *instant of seeing, time for understanding, and moment for concluding* (LACAN, 1989), which gives access to the subjective temporal logic – the subject being analyzed is doing free association before the analyst, who leads them by paralyzing previous knowledge and questioning certainty. There is no possible chronological time for the subject to elaborate a conclusion about their comprehension.

> It is not, therefore, due to some dramatic contingency, or to the gravity of what's at play, or to the emulation of the game, that time is essential; it is under the urgency of the logical movement that the subject precipitates, head first, both his judgment and game, and the etymological sense of the verb, providing the modulation in which the tension of time turns into the tendency to carry out the act that manifests to the others the fact that the subject has concluded[4] (LACAN, 1989, p. 196, our translation).

One concludes that the chronological time imposed by CTT and fulfilled by the role of teacher-who-is-supposed-to-know is incompatible with learning. In genuine listening, one waits for the students to express their thoughts in their own time – the time a singular subject takes to manifest to the other that they have reached a conclusion.

In order to develop the environment in which we will broach the three bridges, we have used *problem-based learning* (PBL) to foster *professional identity*, thus

[4] Original in Spanish: No es pues debido a alguna contingencia dramática, la gravedad de lo que está en juego, o la emulación del juego, por lo que el tiempo apremia; es bajo la urgencia del movimiento lógico como el sujeto precipita a la vez su juicio y su partida, y el sentido etimológico del verbo, la cabeza por delante, de modulación en que la tensión del tiempo se invierte en la tendencia al ato que manifiesta a los otros que el sujeto ha concluido.

preventing students from dropping out of the course, since the processes for identifying with the profession present the engineer as a problem-solver. As has been discussed in the literature about PBL (MILLS; TREAGUST, 2004; PERRENET; BOUHUIJS; SMITS, 2000; RIBEIRO, 2008; JONASSEN, 2014; KOLMOS; GRAAFF, 2014; DAHL, 2018), the teacher is required to organize the didactics so as to overcome the discontinuity between theory and practice, by proposing contextualized situations related to the profession domain, in this case, engineering. A course given under PBL concepts is subject to micro-modifications that lead to a different, multidisciplinary structure. This multidisciplinary character must allow the teacher to go beyond the domain of the technical knowledge, and to take the opportunity to explore the connections between the so-called academic knowledge and professional aspects. It is about enabling the building of the third bridge to address the discontinuity evidenced in the transition from university to professional life, in which real problems are related to sociopolitical and economic issues. It is important to note that although we teach classes in the first four semesters of engineering, and therefore do not have access to the education work that takes place in more specialized subjects toward the end of the program, we still posit that the process of building the third bridge can begin in the calculus classes, thus facilitating its conclusion in the final semesters of the program. The environment in the calculus classes must be conducive to encouraging students to make decisions and justify their choices.

On the other hand, we are aware of restrictive factors. In the UERGS Computer Engineering program, the traditional curriculum for calculus I includes content involving differentiation and anti-differentiation operators; integration is given in calculus II, of which calculus I is a requirement. With this in mind, and accepting the challenge of understanding that the professional object is multidisciplinary, the idea is to introduce calculus to students from a perspective that includes both professional and mathematical signifiers, as in the following examples.

> *Problem with mechanical and electrical quantities.* An electrical test charge is moving at constant speed on an axis on which there's a charge generating an electrical field. The challenge is to analyze the instant power and the energy provided by the field to the test charge.

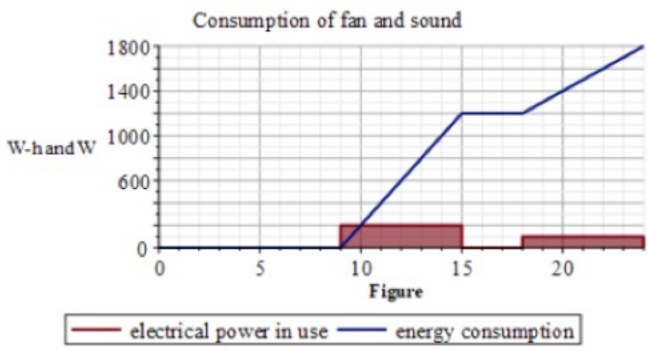
This problem requires students to research signifiers that are part of a computer engineer's education: joule, watt, electric field, power, energy, electric potential, volt, amperes, etc. The difficulty goes beyond identifying what these are; they will have to find mathematics signifiers to develop a solution that uses these concepts. Establishing what these signifiers are requires that students use their everyday language. For instance, "electric field is the force applied on a charge caused by its proximity to other charges," and "that force is measured in newtons per coulomb." In this situation, students are encouraged to research these signifiers both online and by asking physics and engineering professors. These people can help the students learn about the professional practice – in this example, the importance of studying electromagnetism problems for application in developing microprocessors and industrial and residential automation processes in which software is used along with electrical systems. Regarding the mathematical signifiers, the problem involves change rates (derivation) and their accumulations (integration). Also present is the algebraic transformation of an expression, such as force, as a function of a distance variable, in order to turn it into a function of a time variable, an expression of power that provides energy under the integral or anti-differential operators.

This problem allows us to touch on the fundamental theorem of calculus (FTC) before it is normally introduced in the course. A simpler example to indicate the demand on everyday speech, *phoneme*, is to work with a description of the FTC that can be expressed colloquially, such as "the derivative of the integral is the function itself, and the integral of the derivative is the variation of the function."

As a complement, we also have the grapheme and matheme expressions. Learning the FTC implies in the ability to sharpen the phoneme expression by significantly writing the usual graphemes

$$\frac{d}{dt}\int_a^t f(x)\,dx = f(t) \qquad \int_a^b \frac{dF}{dt}(t)\,dt = F(b) - F(a)$$

Learning the FTC also implies a justification in the matheme expression, by presenting a figure as the one shown below, that applies Barrow's seminal idea of using the lengths of line segments to represent areas.



To summarize, we say that learning the FTC means being able to represent oneself by the FTC in the three different expressions: with symbols, graphs, and images.

We must remember that the student's motivation for engaging comes primarily from seeking to increase the value of their professional training. Throughout their college work, first and foremost, students aim to deal with the credit system in order to be approved in the courses and secure a certification at the end of the program. Baldino and Cabral (2015, 2013) clearly characterize the role of the school institution and its relationship to the credit system: it is a production unit in the capitalist system. The school, with its agents taking on institutional roles, produces a merchandise, the *qualified labor force* (QLF) for the market. Over the years in which the subject invests his labor as a student, he works with his peers to produce the certified merchandise; some students may not obtain the diploma that certifies the qualification of the invested capital as QLF recognized by the institution. The diploma is the guarantee that some students were able to collect the work of all those who were not authorized to claim credit for a course. In that sense, we say that the students who were able to recover the capital they invested as QLF participated, throughout the program, in a process of appropriation of surplus value. All students worked, but only those who passed collect the total value produced.

The school determines that students must be "evaluated" by means of written tests, and we recognize it is impossible to eliminate the distinction between success and failure based on merit according to the content given. On the other hand, and using a certain latitude we are afforded by the fact that the institution recognizes our pedagogical authority, we have introduced criteria that are compatible with the maxim "one teaches by listening, and one learns by talking," in an environment of collaboration and cooperation.

It is a known fact that evaluation processes influence the way students study in the various courses, and we use that to our advantage. Pichon-Rivière (1988) states that the learning process requires a change in the subject about the way he deals with knowledge – in our case, knowledge of mathematics –, and that it takes place under an amount of pressure that's neither excessive nor too little. Under that premise, we recognize the pressure imposed by the credit system on the subject who is learning, and we introduce the concept of *understanding* as the students' ability to support their subjective representation, from their point of view, by a signifier, providing a precise basis for *summative evaluation* (BALDINO; CABRAL, 2022).

Our classroom practice is based on the fact that, through dialog, students have to justify what we call *quilting points*, that were previously set out in a worksheet. Studying a worksheet is the effort that can be demanded from the students, for which they receive points that will accumulate to result in either approval or failure in the course. Attributing grades in this manner, by rewarding the effort invested by the students, takes place in class, under situations designed to foster understanding. It is an explicit criterion that can be measured, and the given grade can be contested by the student, since in remote teaching the interactions and dialogs are recorded in video.

The study

This is a qualitative study in the form of a differential intervention that allows a teacher to approach their own classroom as the object of research, becoming a *teacher-researcher* (ATKINSON, 1994; HUILLET; ADLER; BERGER, 2011; KIERAN, KRAINER, SHAUGHNESSY, 2013; LERMAN, 1990; PALLIS, 2009).

The work will be developed with students in the calculus I course, in the second semester[5] of the UERGS program for computer engineering, at the Guaíba campus. The proponents of the project are responsible for the manner in which the classes are given. The computer engineering curriculum is comprised of an expressive number of courses in electronics and computer science. The program requires a minimum of 10 months, during which students must complete 262 credits in courses, internships, and a capstone project. In the end, students who graduate as bachelors in computer engineering must be prepared to enter the workforce in areas that require expertise in hardware and software. Computer engineers

> […] define and coordinate computer systems projects, define and implement computer architectures, computer or electronic device networks, and industrial automation processes; they further propose and implement projects for embedded and real-time systems for industrial, commercial, and scientific applications, work in computer manufacturing plants, and in industries that use automated processes (UERGS, 2022).

The calculus I course is organized into two weekly classes lasting 100 minutes each. From the start of the new Coronavirus pandemic, these classes have taken place via online education. Based on prior experience with online education, classes in the first semester of 2022 will all take place via synchronous videoconference using the

[5] As in other Brazilian universities, in their first semester, freshmen are enrolled in a "Mathematics for Engineering" course (similar to pre-calculus), a 4-credit, 60 hour workload. The calculus I class is offered in the second semester.

Zoom Meetings tool[6]. The university offers the use of Moodle to both teachers and students – a platform that will be used, among other things: (1) to interact with students during the week, (2) as a repository for study and information assets, and (3) to give and turn in assignments.

Classes will consist of conversations with all participants – professors and students – for the entire 100 minutes, with open cameras, thus characterizing the moments of active participation and of *cooperative and collaborative work*. Accordingly, all activities proposed in class will follow the concept of *problem-based learning*. In regard to building the three bridges, the following will be part of the discussions: (1) problems related to signifiers located in the professional domain and required from a bachelor in computer engineering; (2) basic mathematical signifiers in problems, as part of higher-education requirements; and (3) how the previous two items relate and fit in with their professional education.

Since classes will take place over Zoom Meetings, their videos will be recorded. In order to enable the constant adjustment process required in this course, videos from one class may be used to evaluate the work performed by both students and professors, to define the classroom activities for the next class, and to assign grades to student participation. Adjusting the classroom work from one class to the next is also informed by a discussion between the teacher-researchers who participate in all moments that define the educational practice. The professors will, whenever possible, record their in-class observations at the end of each class, thus creating a "field log."

Resources

The professors already own:

- 2 new computers: a Dell PC and an Apple MacBook Air,
- 2 new Apple iPads – a 12.9" and an iPad mini,
- 2 Epson printers,
- Licenses for various proprietary software, including: Maple, CorelDraw, Office 365, and Office MacOS,
- A Zoom Meetings use license,

[6] The institution offers the Google for Education Suite for both faculty and students. However, to date, Google's videoconference tool does not offer the same features as Zoom Meeting, which has allowed us to use an iPad as a digital surface – a digital whiteboard shared with the students – thus facilitating the conversation supported by mathematical writing. It works very differently from Google Meet's interactive whiteboard.

– A 300 Mbps fiber optic internet service by Vivo,
– A physical library and a virtual library (Kindle).

Resources offered by the institution:

– Free software for supporting learning in a virtual environment, Moodle,
– Google for Education suite,
– Pearson virtual library (BV).

Timetable and expectations

The intervention in the classroom proposed by this project will begin in the first day of this school semester: March 8, 2022. On July 8, the course will end. The in-class activities, the answers given by the students to the interventions, their participations recorded in video, and the field log, all of which will be collected over the semester, will be made available in reports prior to the writing of the articles. The worksheets and all assets created and used throughout the semester will be made available under a Creative Common license[7], initially at the professors' website, https://cabraldinos.mat.br/. Later, after adding considerations about the work done, the researchers intend to compile all these assets into an educational product and publish it in the repositories that the institution is organizing for, among other purposes, its professional master's programs.

We hope that this research will foster contacts with other researchers, some of whom were mentioned here, possibly leading to collaborations. We aim to produce at least one article for publication in an indexed journal, and a report to be presented at the *13th Congress of the European Society for Research in Mathematics Education* (CERME13), which will take place from January 13 to February 4, 2023, in Budapest. Finally, since one of the researchers is a thesis advisor in the Professional Master's Program in Education for the Sciences, Technology, Engineering, and Mathematics at UERGS University, Guaíba campus, we expect one or more of the program's students may be involved in the project.

[7] An organization that offers sharing of knowledge and educational assets under CC licenses, which are legal free instruments.

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